Energy balance approach for oscillator parameter ID

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Outline

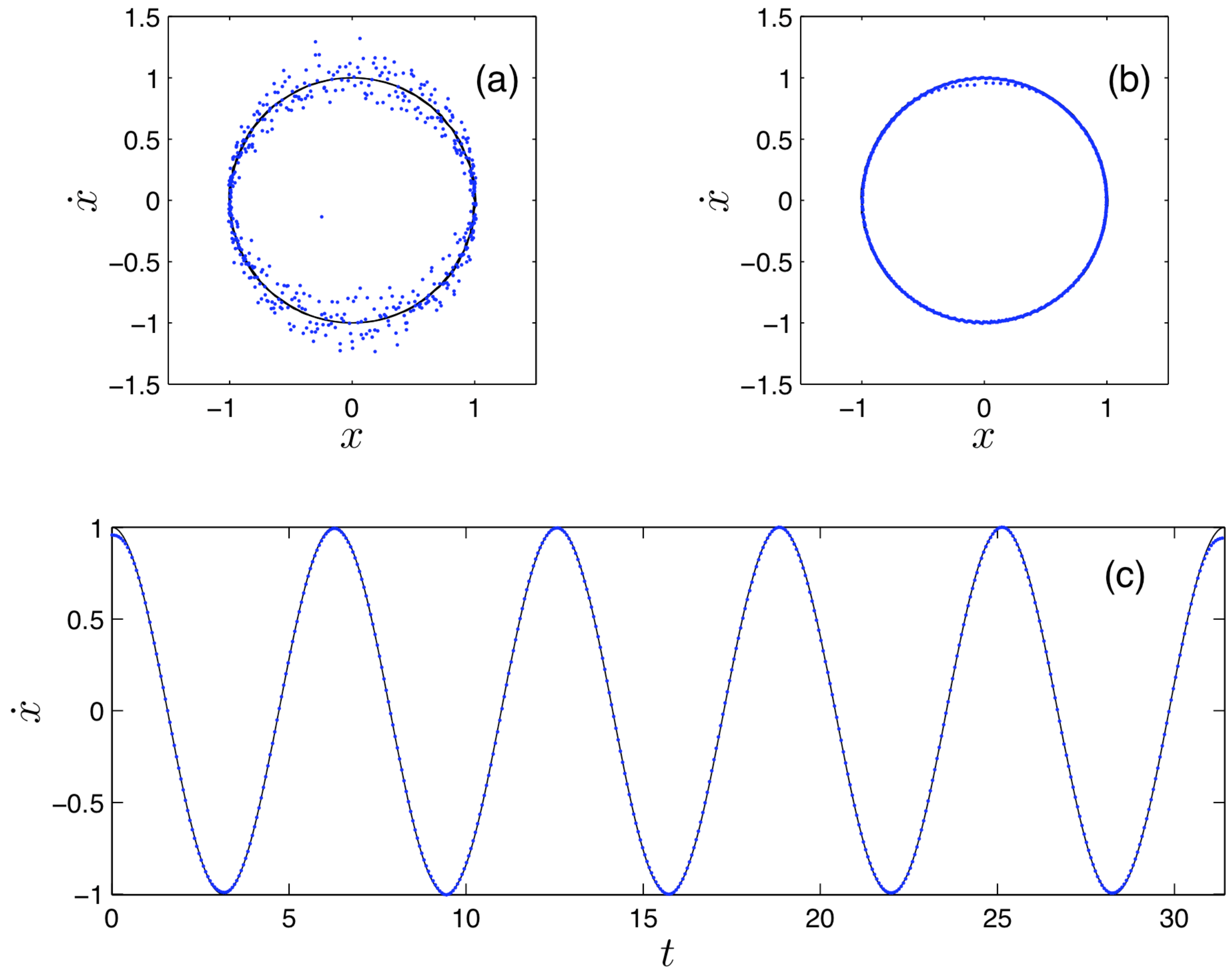
1. Background
2. Energy balance ID
3. Case studies
4. Summary

Select prior work

- 1992 - Mohammad, Worden, Tomlinson, Direct approach
- 1988 – Yashuda, Harmonic balance ID
- 1992 – Yiang and Feeny, ID from Chaotic response
- 2003 – Nichols and Virgin, ID from Chaotic interrogation
- 2006 – Liang and Feeny, Energy balance (friction)

Noisy data: Splines or smoothing splines

$$x(t) = b_{k0} + b_{k1}(t - t_k) + b_{k2}(t - t_k)^2 + b_{k3}(t - t_k)^3$$



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Example energy balance: Oscillator

Example system

$$m\ddot{x} + c\dot{x} + kx + k_3x^3 = F\cos\Omega t$$

Balance energy

$$\int_{t_1}^{t_2} \left(m\ddot{x} + c\dot{x} + kx + k_3x^3\right)\dot{x}dt = \int_{t_1}^{t_2} \dot{x}F\cos\Omega t\, dt$$

Generic expression

$$T_{1\to 2} + U_{1\to 2} = W_{in} - W_d$$

Energy balance

Energy balance

$$T_{1\to 2} + U_{1\to 2} = W_{in} - W_d$$

Conservative terms

$$T_{1\to 2} = \frac{1}{2} m \left(\dot{x}(t_2)^2 - \dot{x}(t_1)^2 \right)$$

$$U_{1\to 2} = \frac{1}{2} k \left(x(t_2)^2 - x(t_1)^2 \right) + \frac{1}{4} k_3 \left(x(t_2)^4 - x(t_1)^4 \right)$$

Nonconservative terms

$$W_d = \int_{t_1}^{t_2} c\dot{x}^2 \, dt \qquad W_{in} = \int_{t_1}^{t_2} \dot{x} F \cos \Omega t \, dt$$

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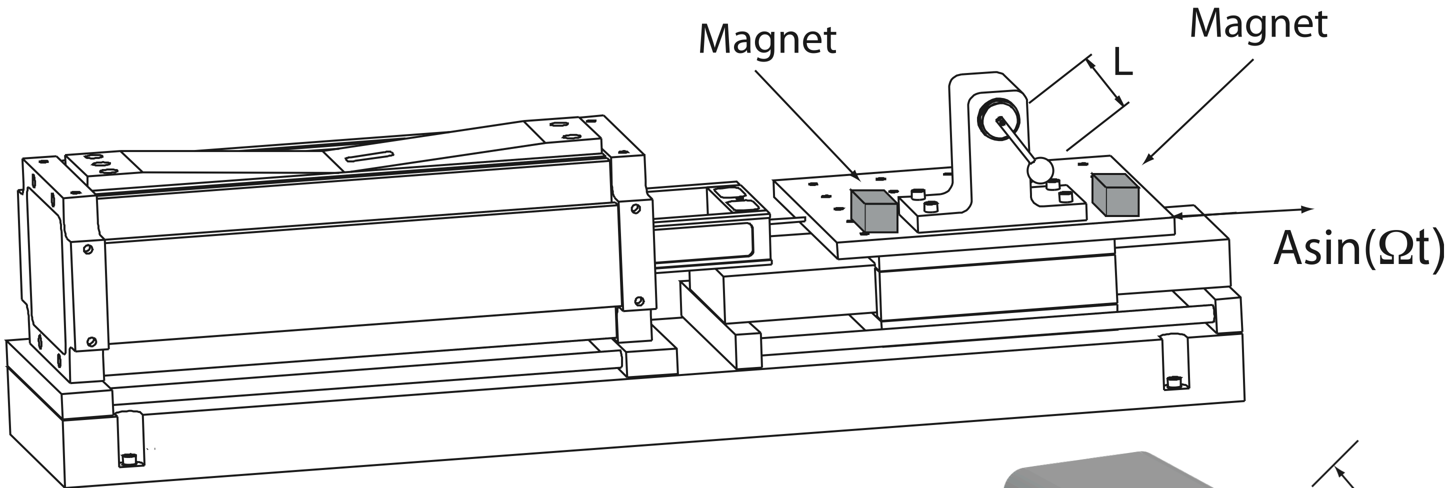
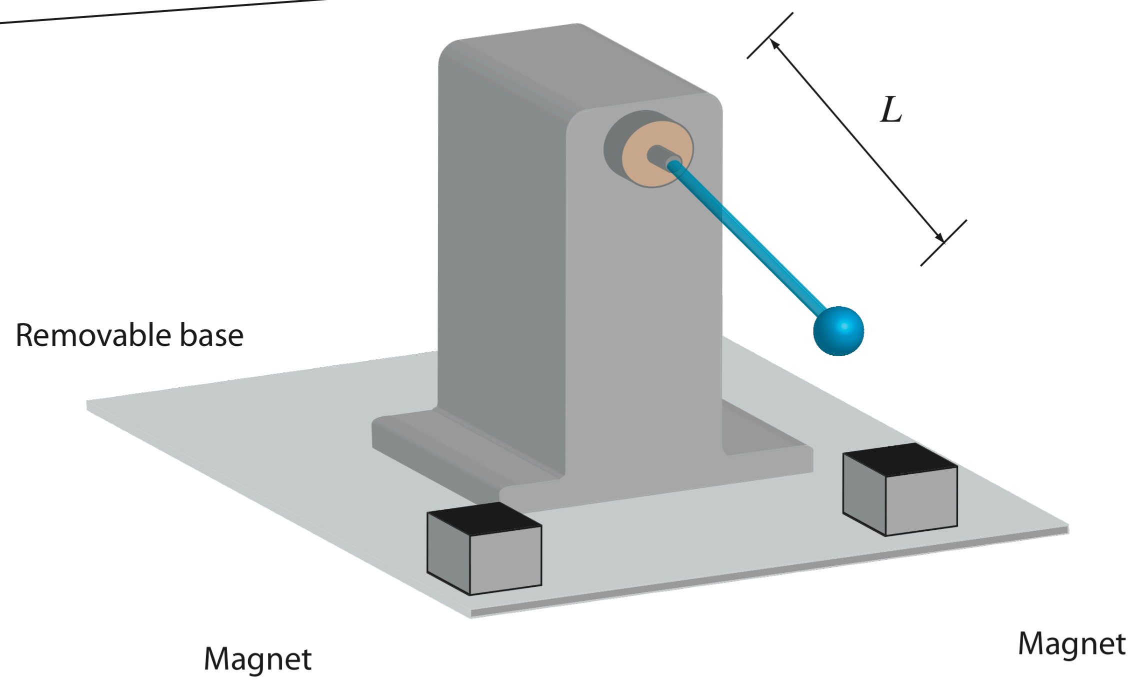
Experimental system





Compare: Energy ID and analytical soln

Governing eqn

$$\ddot{\theta} + 2\mu\omega\dot{\theta} + \omega^2 \sin\theta$$

$$\ddot{\theta} + \omega^2\theta = -f\left(\theta, \dot{\theta}\right)$$

Assumed soln

$$\theta(t) = a\cos\left(\omega t + \phi\right) = a\cos\psi$$

Avg eqns

$$\dot{a} = \frac{1}{2\pi\omega}\int_0^{2\pi} \sin\psi f\left(a\cos\psi, -a\omega\sin\psi\right) d\psi = -\mu\omega a$$

$$\dot{\phi} = \frac{1}{2\pi\omega a}\int_0^{2\pi} \cos\psi f\left(a\cos\psi, -a\omega\sin\psi\right) d\psi = \frac{3a^2\beta}{8\omega}$$

Approx analytical solution

Integrated terms

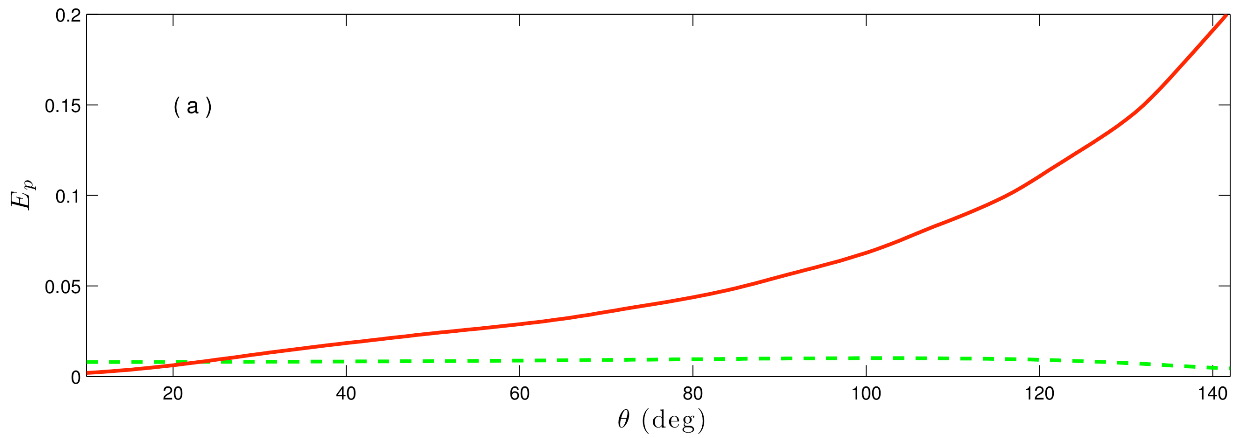
$$a = a_0 e^{-\mu\omega t}$$

$$\phi = \frac{a_0^2}{32\zeta}\left(e^{-2\mu\omega t} - 1\right) + \phi_0$$

Analytical soln

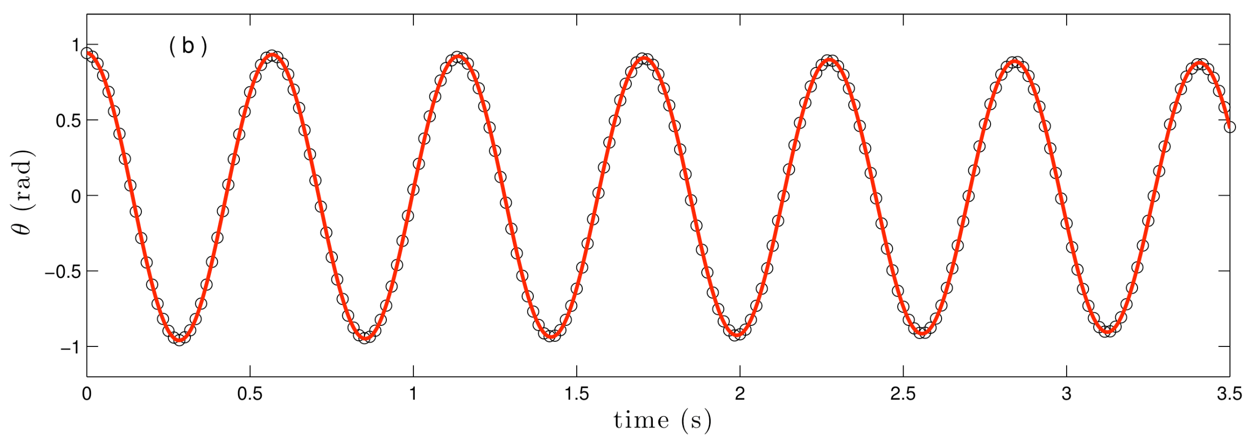
$$\theta(t) = \vartheta_0 e^{-\mu\omega t} \cos\left(\omega t + \frac{\vartheta_0^2}{32\mu}\left(e^{-2\mu\omega t} - 1\right)\right)$$

Comparison: Analytical vs energy



Error norm

$$E_p = \sqrt{\frac{1}{n_p}\left(\left(\frac{\mu_e - \mu}{\mu}\right)^2 + \left(\frac{\omega_e - \omega}{\omega}\right)^2\right)}$$



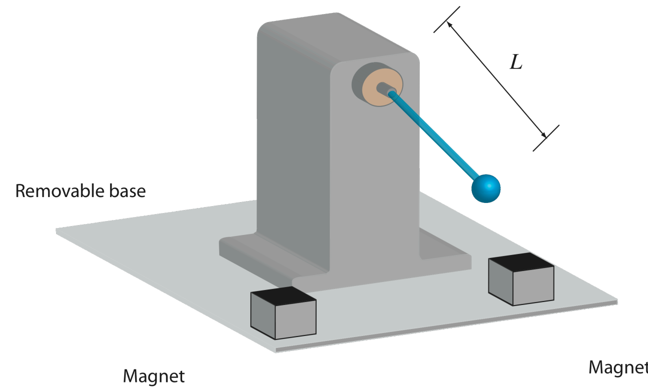
Fit analytical soln

$$E_T = \sum_{k=1}^{N} (\theta_m(t_k) - \theta(t_k))^2$$

Magnetic pendulum example

Governing eqn

$$\ddot{\theta} + 2\mu\omega\dot{\theta} + \omega^2 \sin\theta + \sum_{n=1}^{3} \hat{\alpha}_n (n+1)\theta^n = 0$$



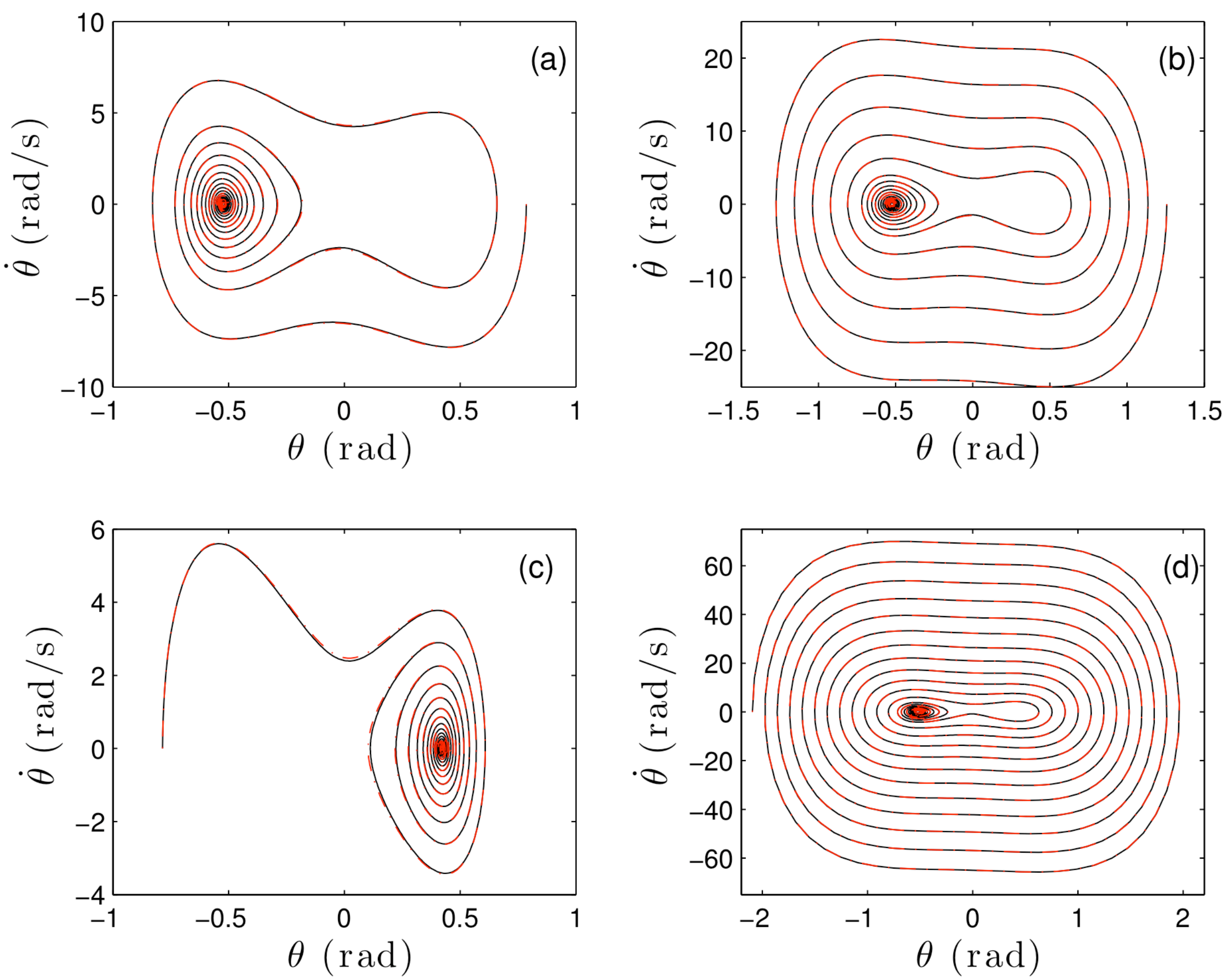
Error norm

$$E_p = \sqrt{\frac{1}{n_p}\left(\left(\frac{\mu_e - \mu}{\mu}\right)^2 + \left(\frac{\omega_e - \omega}{\omega}\right)^2 + \sum_{n=1}^{3}\left(\frac{\alpha_{ne} - \alpha_n}{\alpha_n}\right)^2\right)}$$

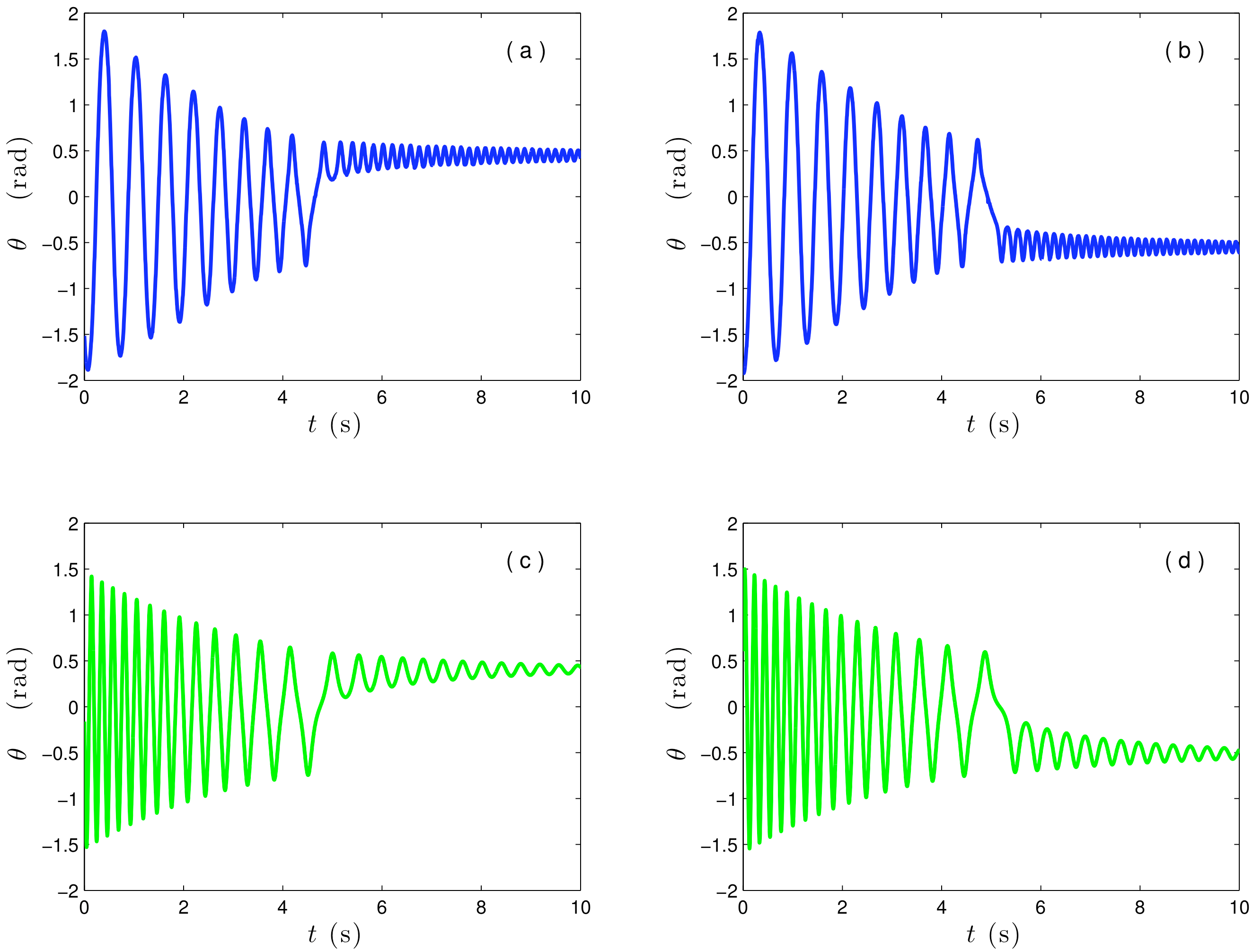
Steps

-Test energy balance on synthetic data
- Apply to experimental system

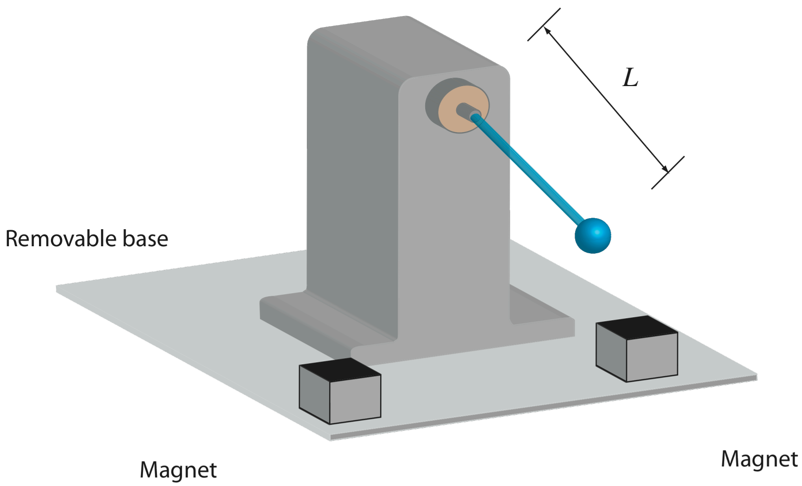
Comparisons for simulated data



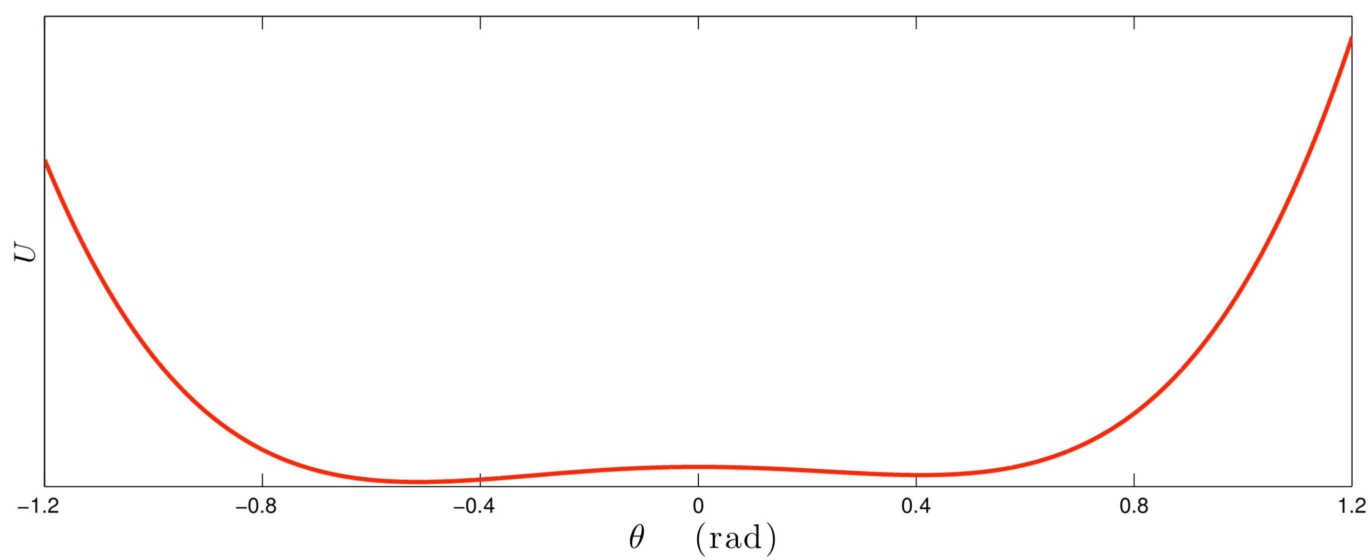
Experimental comparisons



Experimental bistable potential wells



Parameter	μ	ω (rad/s)	α_1 (N/rad)	α_2 (N/rad^2)	α_3 (N/rad^3)
Reference	0.0509	11.74	-138.3	21.3	163.1
Estimated	0.0510	11.71	-137.3	21.3	162.9
Experiment	0.0287	11.74	-145.25	25.74	188.11



Forced oscillator example

Example system

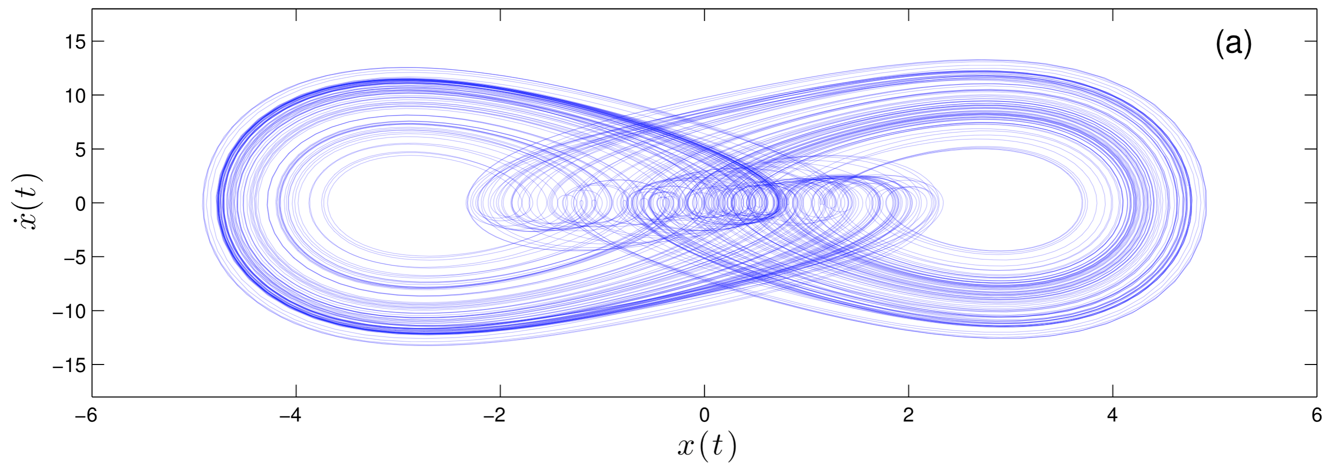
$$m\ddot{x} + c\dot{x} + kx + k_3 x^3 = F\cos\Omega t$$

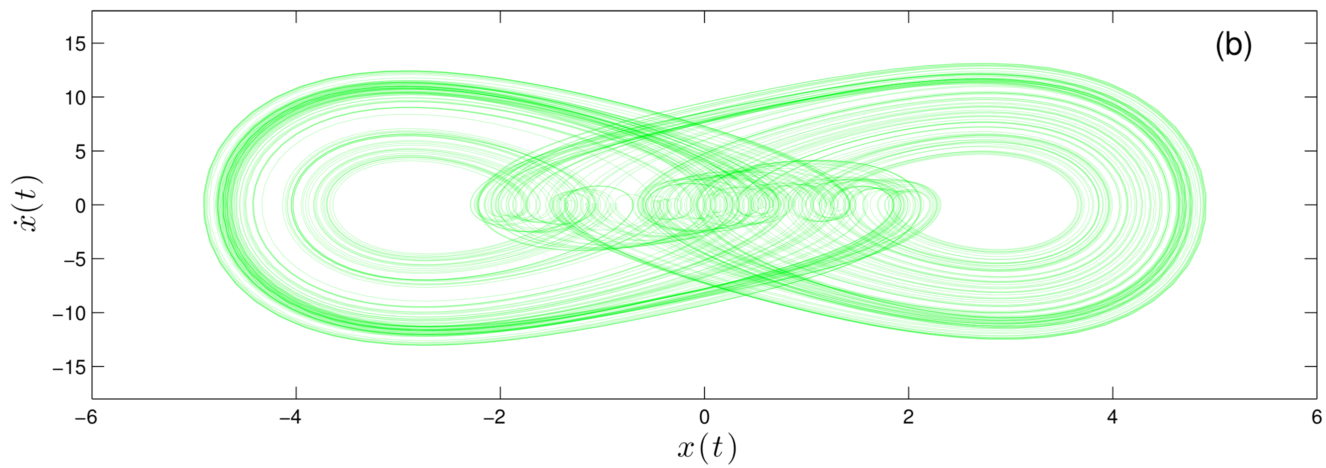
Work into system

$$W_{in} = \int_{t_1}^{t_2} \dot{x} F \cos\Omega t \, dt$$

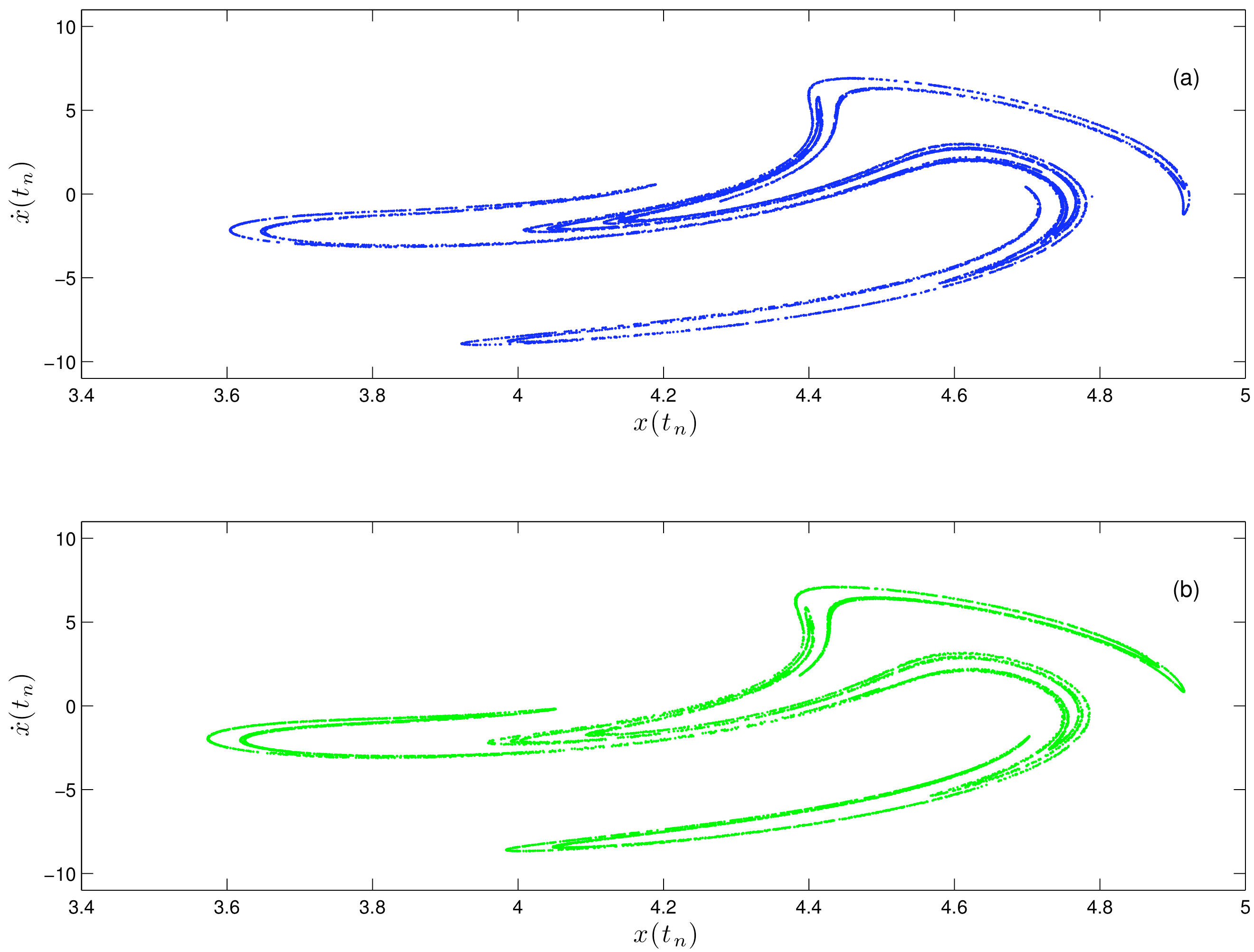
$$x(t) = b_{k0} + b_{k1}(t - t_k) + b_{k2}(t - t_k)^2 + b_{k3}(t - t_k)^3$$

Comparison of phase plane trajectories





Comparison of Poincare Sections



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Questions ?